Problem 05: Division

Given two natural numbers a and b, compute the quotient and remainder of a/b, using only subtraction, addition, and the comparision operators.

$$\begin{array}{lcccccccc} A & = & \mathbb{N} & \times & \mathbb{N} & \times & \mathbb{N} & \times & \mathbb{N}_0 \\ & & a & & b & & d & & r \\ B & = & \mathbb{N} & \times & \mathbb{N} & & & & \\ & & a' & & b' & & & & \\ Q & = & (a' = a) \wedge (b' = b) & & & & & & \\ R & = & Q \wedge (r < b) \wedge (a = db + r) & & & & & & \end{array}$$

Solution

Since only subtraction and addition are allowed, we'll set r to a and subtract b from it in every iteration, while increasing d:

$$\begin{array}{lcl} P & = & Q \wedge (a = db + r) \\ \neg\pi & = & r < b \\ \pi & = & r \geq b \\ t & = & a - db \\ Q' & = & Q \wedge (d = 0) \wedge (r = a) \\ \\ P^{d \leftarrow (d+1), r \leftarrow (r-a)} & = & Q \wedge (a = (d+1)b + (r-a)) \\ & \simeq & P \wedge \pi \end{array}$$

The resulting program: