Problem 25: Sum of Digits

Given two natural numbers x and k, compute the sum of digits representing x in base-k.

$$
\begin{array}{rcl}
A & = & \underset{x}{\mathbb{N}} \times \underset{k}{\mathbb{N}} \times \underset{s}{\mathbb{N}} \;|\!\times\; \underset{y}{\mathbb{N}} \\
B & = & \underset{x'}{\mathbb{N}} \times \underset{k'}{\mathbb{N}} \\
Q & = & (x' = x) \wedge (k' = k) \\
R & = & Q \wedge s = \sum_{i=0}^{\lfloor \log_k x \rfloor} (x \bmod k^{i+1}) \operatorname{div} k^i
\end{array}
$$

Solution

This problem is a lot like number 24, so no extra explanation should be necessary.

$$
\begin{array}{lcl}
P & = & Q \wedge s = \sum_{i=0}^{\lfloor \log_k x \rfloor - \lfloor \log_k y \rfloor} (x \bmod k^{i+1}) \operatorname{div} k^i \\
\neg\pi & = & y < k \\
\pi & = & y \geq k \\
t & = & \lfloor \log_k y \rfloor + 1 \\
Q' & = & Q \wedge (y = x) \wedge (s = x \bmod k)
\end{array}
$$

$$
\begin{array}{lcl}
P^{y \leftarrow (y \operatorname{div} k)} & = & Q \wedge (s = \sum_{i=0}^{\lfloor \log_k x \rfloor - \lfloor \log_k (y \operatorname{div} k) \rfloor} (x \bmod k^{i+1}) \operatorname{div} k^i) \\
& = & Q \wedge (s = \sum_{i=0}^{\lfloor \log_k x \rfloor - \lfloor \log_k y \rfloor} (x \bmod k^{i+1}) \operatorname{div} k^i + y \bmod k) \\
& \simeq & P \wedge \pi \wedge (s = s + y \bmod k)
\end{array}
$$

The resulting program:

$y, s := x, (x \bmod k)$
$y \geq k$
$s := s + y \bmod k$
$y := y \operatorname{div} k$